

Math 109 HW11

Fall 2018

1. Determine whether the series converges or diverges using the Ratio Test.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{2^n n^2}$$

$$\sum_{n=1}^{\infty} \frac{n!}{10^n}$$

2. Determine whether the series converges or diverges using the Root Test.

$$\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n(n+1)} \right)^n$$

$$\sum_{n=1}^{\infty} \frac{(-2)^n + 5^n}{n^n}$$

$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n} \right)^{n^2}$$

3. For which of the following series is the Ratio Test inconclusive (that is, it fails to give a definite answer)?

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1}$$

4. Find the radius of convergence and interval of convergence of the following series.

$$\sum_{n=1}^{\infty} n!(5x - 4)^n$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x - 2)^n}{3^n n^2}$$

5. Find a power series representation for the following functions.

$$f(x) = \frac{x}{x^2 + 2}$$

$$f(x) = \frac{x + a}{x^2 + a^2}, a > 0$$

6. Find a power series representation for the following functions and determine the radius of convergence.

$$f(x) = \ln(1 - x^2)$$

$$f(x) = x^2 \tan^{-1}(x^3)$$

7. Evaluate the indefinite integral as a power series and determine the radius of convergence.

$$\int \frac{t}{1 - t^4} dt$$

$$\int \frac{\tan^{-1} t}{t} dt$$