

AS.110.109: Calculus II (Eng)

Chapter 7: Integrals

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About this course

- Instructor: Yi Wang MWF
- Office hours: MW 9AM-10AM,
- TA office hours: see course website
- Course Website: <http://www.math.jhu.edu/math109/>
- Textbook: Single Variable Calculus: Early Transcendentals, 8th Edition. James Stewart
- 11-12 Homework assignments, posted on the course website, due at Sections
- Midterm 1, Midterm 2, Final
- Registration information of Pilot program
- Help Room (Krieger 213) Schedule

7.1: Integration by parts

- Product rule: suppose f and g are differentiable, then

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x).$$

Integrate both sides,

$$f(x)g(x) = \int f'(x)g(x)dx + \int f(x)g'(x)dx.$$

- Rearranging terms, we get **Integration By Parts formula**

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx.$$

When to apply integration by parts?

- When the integrand can be written as product $f(x)g'(x)$.
 - ▶ For example, $\int x \sin x dx$, $\int e^x x dx$, etc.
- How to remember the **Integration By Parts (IBP) formula**? Let $f(x) = u$, $g(x) = v$. Hence $du = f'(x)dx$, $dv = g'(x)dx$. Then IBP formula is

$$\int u dv = uv - \int v du.$$

Example 1

- Integrate $\int xe^x dx$. It is not anything that we know from the chart on page 471. But we know

$$\int x dx = \frac{1}{2}x^2 + C,$$

$$\int e^x dx = e^x + C.$$

Example 1

- We have two ways to choose f and g .
 - ▶ Choice 1: $f(x) = x$, $g'(x) = e^x$. Or
 - ▶ Choice 2: $f(x) = e^x$, $g'(x) = x$.

But in this example, only the first choice works.

General criteria to choose f and g : factorize the integrand into $f(x) \cdot g'(x)$, so that

1. it is easy to integrate $g'(x)$;
2. $f'(x)$ is simpler, in the way that $f'(x)g(x)$ is easy to integrate.

Example 1

Choice 1: Choose $f(x) = x$, $g'(x) = e^x$. Then both two criteria are satisfied. $g(x) = \int e^x dx = e^x$, (choose $C = 0$) and $f'(x)g(x) = e^x$ is easy to integrate.

$$\begin{aligned}\int x \sin x dx &= \int f(x)g'(x)dx \\ &= f(x)g(x) - \int f'(x)g(x)dx \\ &= xe^x - \int 1 \cdot e^x dx \\ &= xe^x - \int e^x dx \\ &= xe^x - e^x + C.\end{aligned}\tag{1}$$

Example 1

- What would happen if we adopt Choice 2? Then it satisfies the first criterion but not the second. This will make the computation more complicated after we do integration by parts:

$$\int xe^x dx = e^x \cdot \left(\frac{1}{2}x^2\right) - \int \left(\frac{1}{2}x^2\right) \cdot e^x dx. \quad (2)$$

See we get stuck, as we don't know how to integrate the second term $\int \left(\frac{1}{2}x^2\right) \cdot e^x dx$.

Reading assignments

Reading assignments: Review integration table on Page 471 and substitution (section 5.5 in this book, or you can refer to the textbook you learned before on this subject.)
Review product rule, chain rule and quotient rule of differentiation.